2.2

Function Sense

Translating Functions

LEARNING GOALS

In this lesson, you will:

- Analyze the basic form of a quadratic function.
- Identify the reference points of the basic form of a quadratic function.
- Understand the structure of the basic quadratic function.
- Graph quadratic functions through transformations.
- Identify the effect on a graph by replacing $f(x)$ by $f(x - C) + D$.
- Identify transformations given equations of quadratic functions.
- Write quadratic functions given a graph.

KEY TERMS

- reference points
- transformation
- rigid motion
- argument of a function
- translation

Have you ever taken a road trip? For most American children, some road trips were the bane of existence—especially with annoying siblings. Of course, on the drive back, kids might get excited by a point of reference such as a road sign indicating the number of miles remaining before getting home, or seeing a landscape that is comfortably familiar. Adults commonly use reference points on road trips as well. Most U.S. national map books give estimated hours of travel and distances between key cities across the country. These help motorists and truckers determine if they should continue on, or get off the highway for a bit of shut eye.

What types of reference points have you used? How did you use those reference points?

PROBLEM 1 It All Comes Down to the Basics

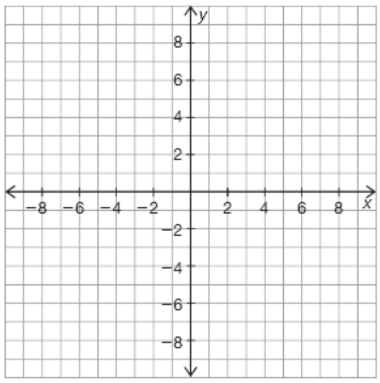
So far in this course, all the different forms of quadratic functions you have studied have been based on the function $f(x) = x^2$. This function is the basic form of a quadratic function. From the form of this function, you know the vertex is (0,0).

The pattern of this function is that for every input value, x, the output value, $f(x)$, is squared.

1. Let's consider the structure of $f(x) = x^2$ and its corresponding graph.

   a. Complete the table. Then plot and label the points on the coordinate plane.

x	$f(x) = x^2$
0	
1	
2	

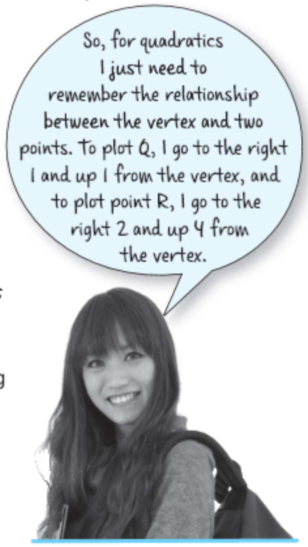


   b. Draw a dashed line to represent the axis of symmetry. Then plot and label all symmetric points. Finally, draw a smooth curve to represent $f(x) = x^2$.

PROBLEM 2 Up, Down, Left, Right

You just analyzed the basic form of a quadratic function, $f(x) = x^2$. For a quadratic function, if you know the vertex and any two points to the right of that vertex, you can use the axis of symmetry to identify the other half of the parabola. A set of key points that help identify the basic form of any function are called **reference points**. The reference points of the basic quadratic function are defined in the table shown.

Reference Points of the Basic Quadratic Function	
P	(0, 0)
Q	(1, 1)
R	(2, 4)



Now that you understand the structure of the basic quadratic function, let's explore how to apply *transformations* to graph new functions. Recall that a **transformation** is the mapping, or movement, of all the points of a figure in a plane according to a common operation. Translations, reflections, rotations, and dilations are examples of transformations. In previous courses, you studied the effects that transformations had on graphs of functions and various figures on the coordinate plane. A **rigid motion** is a transformation that preserves size and shape.

1. Identify which transformations are rigid motions that preserve size and shape.

Previously, you worked with the vertex form of quadratic functions, $f(x) = a(x - h)^2 + k$, which represent transformations of $f(x) = x^2$.

2. Given a quadratic function written in vertex form, $f(x) = a(x - h)^2 + k$, identify the effect the h- and k-values have on the graph of the basic quadratic function.

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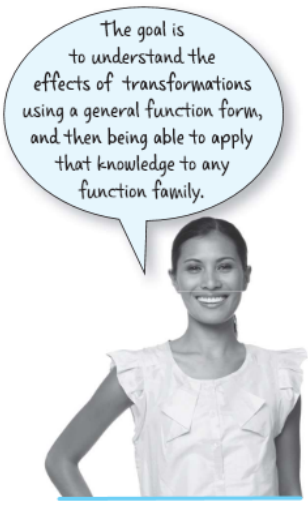
Vertex form is a specific form of a function in the quadratic function family. In this lesson, you will use the basic quadratic function to explore various function transformations. Eventually, you will learn how to generalize about transformations performed across many different function types, but first let's establish a form that will be representative of any function. Transformations performed on any function $f(x)$ to form a new function $g(x)$ can be described by the transformational function form:

$$g(x) = Af(B(x - C)) + D$$

where A, B, C, and D represent different constants.

Let's make some connections and compare the vertex form of a quadratic function, $f(x) = a(x - h)^2 + k$, to the transformational function form $g(x) = Af(B(x - C)) + D$, where $B = 1$.

3. How are the h- and k-values of the vertex form of the quadratic function represented in the transformational function form?



Let's consider the constants *C* and *D*, where *A* and *B* both equal 1 and the effects each value has on the graph of the basic quadratic function. Let *A* and *B* both equal 1.

Given $f(x) = x^2$

Graph $g(x) = f(x - 4) + 2$

In the function $g(x)$, $C = 4$ and $D = 2$. The *C*-value and the *D*-value will tell you how to translate the function. Notice, the value of 4 is on the *inside* of the function, or the *argument of the function*. The **argument of a function** is the variable, term, or expression on which the function operates. The value 2 is on the *outside* of the function. Recall values on the inside of a function affect the *x*-values of the function, and values on the outside affect *y*-values of the function. So, the *C*-value will tell you how many units to translate the function left or right, and the *D*-value will tell you how many units to translate the function up or down.

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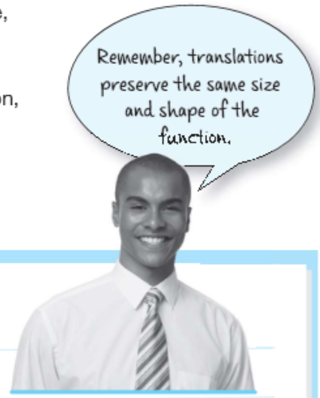
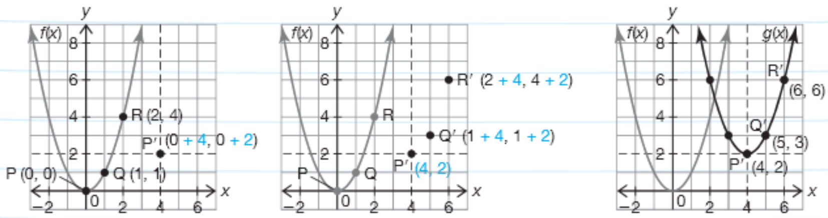


Given $f(x) = x^2$

Graph $g(x) = f(x - 4) + 2$

You can use reference points for $f(x)$ and your knowledge about transformations to graph the function $g(x)$.

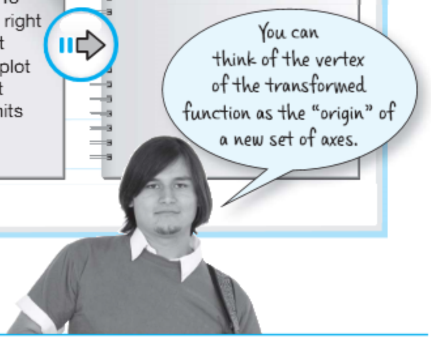
From $g(x)$, you know that $C = 4$ and $D = 2$ which tells you the entire graph will translate 4 units to the right and 2 units up.



The function $f(x)$ is shown with its reference points. To begin to graph $g(x)$, plot the new vertex, (*C*, *D*). This point establishes the new set of axes.

Next, think about the pattern of the basic quadratic function. To plot point Q′, move right 1 unit, and up 1 unit from the vertex. To plot point R′, move right 2 units, and up 4 units from the vertex.

Finally, use symmetry to complete the graph.



4. Analyze the worked example.

a. Complete the table of values to verify the graph.

Reference Points of Basic Quadratic Function		→	Apply the Transformations	Corresponding Points on $g(x)$	
P	(0, 0)	→	(0 + ____, 0 + ____)	P'	(4, 2)
Q	(1, 1)	→	(1 + ____, 1 + ____)	Q'	(5, 3)
R	(2, 4)	→	(2 + ____, 4 + ____)	R'	(6, 6)

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b. Why is it important to establish a new set of axes?

c. Use C and D to write the equations that correspond to the new set of axes.

d. Name the two points symmetric to Q' and R'.

e. The function $g(x) = f(x - 4) + 2$ is written in terms of $f(x)$. Rewrite $g(x)$ in terms of x by substituting $(x - 4)$ for x into $f(x)$, and adding 2 onto $f(x)$.

5. Given $f(x) = x^2$, graph $h(x) = f(x - 2)$.

a. Identify the C- and D-values.

b. Complete the table.

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $h(x)$
(0, 0)	$\rightarrow$	
(1, 1)	$\rightarrow$	
(2, 4)	$\rightarrow$	

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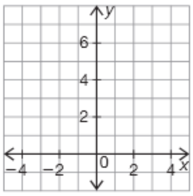
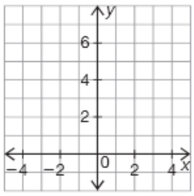
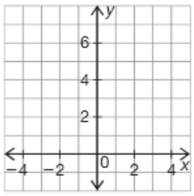
c. Graph $h(x)$ in the same 3 steps as the worked example. Provide the rationale you used below each graph.







A **translation** is a type of transformation that shifts an entire figure or graph the same distance and direction.

Compared with the graph of $y = f(x)$, the graph of $y = f(x - C) + D$:

- Shifts left C units if $C < 0$.
- Shifts right C units if $C > 0$.
- Shifts down D units if $D < 0$.
- Shifts up D units if $D > 0$.

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8. Given $y = f(x)$, write the coordinate notation represented in $y = f(x - C) + D$.

$(x, y) \rightarrow$ ______________________________

Be prepared to share your solutions and methods.

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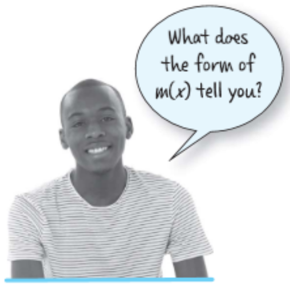
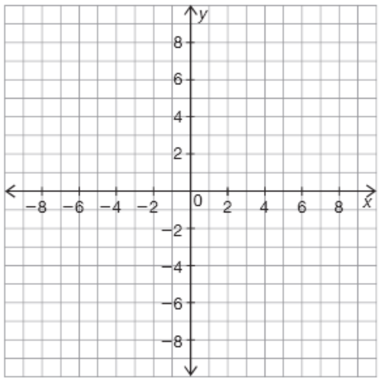
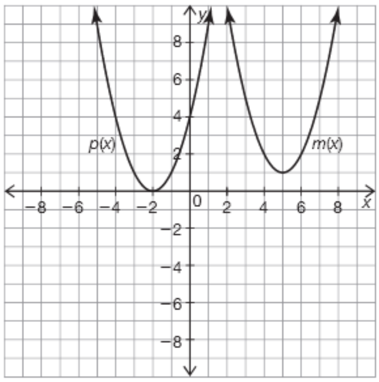
6. Given $f(x) = x^2$, graph the function $m(x) = f(x + 2) - 3$.



Reference Points on *f*(*x*)	→	Corresponding Points on *m*(*x*)
(0, 0)	→	
(1, 1)	→	
(2, 4)	→	



7. Analyze the graphs of $p(x)$ and $m(x)$.



a. If $f(x) = x^2$, write $m(x)$ in terms of $f(x) = x^2$.

b. If $f(x) = x^2$, write $p(x)$ in terms of $f(x) = x^2$.

c. Write $p(x)$ in terms of $m(x)$.

d. Write $m(x)$ in terms of $p(x)$.